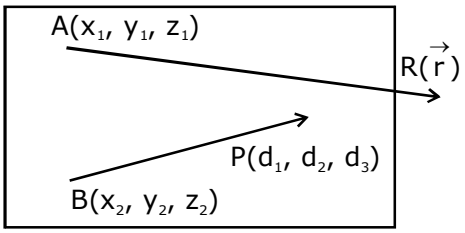
EXERCISE – II

HINTS & SOLUTIONS

Sol.1 A,B



$[\vec{AR}\ \vec{AB}\ \vec{P}] = 0$

$$\begin{vmatrix} x-x_1 & y-y_1 & z-z_1 \\ x_2-x_1 & y_2-y_1 & z_2-z_1 \\ d_1 & d_2 & d_3 \end{vmatrix} = 0$$

or $$\begin{vmatrix} x-x_2 & y-y_2 & z-z_2 \\ x_1-x_2 & y_1-y_2 & z_1-z_2 \\ d_1 & d_2 & d_3 \end{vmatrix} = 0$$

Sol.2 A,C

$x + y + z - 1 = 0$ & $4x + y - 2z + 2 = 0$

put $z = 0$

$x + y = 1$

$4x + y = -2 \Rightarrow x = -1, y = 2$

Point $(-1, 2, 0)$

$$\text{Direction} = \begin{vmatrix} \hat{i} & \hat{j} & \hat{k} \\ 1 & 1 & 1 \\ 4 & 1 & -2 \end{vmatrix}$$

$= \hat{i}(-2-1) - \hat{j}(-2-4) + \hat{k}(1-4)$

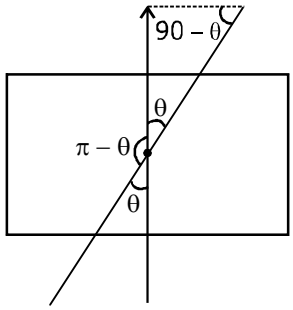
$= -3\hat{i} + 6\hat{j} - 3\hat{k} = -3(1, -2, 1)$

Equation of line in symmetrical form

$$\frac{x+1}{1} = \frac{y-2}{-2} = \frac{z-0}{1}$$

(C) will also satisfy

Sol.3 C,D

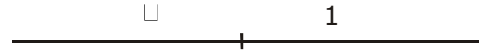


$$\text{Normal vector} = \begin{vmatrix} \hat{i} & \hat{j} & \hat{k} \\ 2 & 3 & -1 \\ 1 & -1 & 1 \end{vmatrix} = 5(1, -1, -1)$$

$$\cos(90-\theta) = \frac{2+2-1}{\sqrt{9}\sqrt{3}} = \frac{1}{\sqrt{3}}$$

$$\sin\theta = \frac{1}{\sqrt{3}} \quad \Rightarrow \quad \cot\theta = \sqrt{2}$$

Sol.4 A,C



$$R\left[\frac{27\alpha+12}{\alpha+1}, \frac{-9\alpha+4}{\alpha+1}, \frac{18\alpha+8}{\alpha+1}\right]$$

Put R in given sphere

$$\left(\frac{27\alpha+12}{\alpha+1}\right)^2 + \left(\frac{-9\alpha+4}{\alpha+1}\right)^2 + \left(\frac{18\alpha+8}{\alpha+1}\right)^2 = 504$$

$\Rightarrow \alpha = 2/3$ internally

$\alpha = -2/3$ externally

Sol.5 A,D

$$\frac{x-1}{2} = \frac{y+3}{1} = \frac{z+1}{-2}$$

Direction of line $\vec{b} = (2, -1, -2)$

(A) Normal of plane $\vec{n} = (2, 2, 1)$

$\vec{b} \cdot \vec{n} = 4 - 2 - 2 = 0$

(B) $\vec{b} \cdot \vec{n} = 2 - 2 + 4 = 4$

(C) $\vec{b} \cdot \vec{n} = 4 + 2 - 2 = 4$

(D) $\vec{b} \cdot \vec{n} = 2 + 2 - 4 = 0$

Sol.6 A,B,C

G(3, 2, 1)

$\overrightarrow{OG} = (3, 2, 1)$

$\overrightarrow{BF} = (3, -2, 1)$

$$\cos\alpha = \frac{(3,2,1).(3,-2,1)}{\sqrt{14}\ \sqrt{14}}$$

$$\cos\alpha = \frac{3}{7} \Rightarrow \alpha = \cos^{-1}\frac{3}{7}$$

Similarly ratate the length 2 get all angle.

Sol.7 C

$x = y = -z$

DR'S $(1,1,-1) = 0$

$\Rightarrow (2, 3, 5).\ (1, 1, -1) = 0$

$2\ (1, 2, 3).\ (1, 1, -1) = 0$

Sol.8 B,D

$$\text{DC'S} = \frac{(\ell_1+\ell_2)\hat{i}+(m_1+m_2)\hat{j}+(n_1+n_2)\hat{k}}{\sqrt{(\ell_1+\ell_2)^2+(m_1+m_2)^2+(n_1+n_2)^2}}$$

$$|\vec{n}| = \sqrt{(\ell_1+\ell_2)^2+(m_1+m_2)^2+(n_1+n_2)^2}$$

$$= \sqrt{2+2\ell_1\ell_2+m_1m_2+n_1n_2}$$

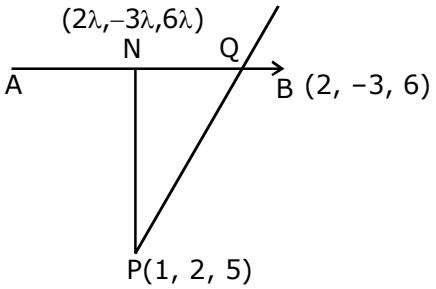
$$\cos\theta = \ell_1\ell_2+m_1m_2+n_1n_2$$

$$|\vec{n}| = \sqrt{2+2\cos\theta} = 2\cos\frac{\theta}{2}$$

angle is $\pi - \theta$

$$|\vec{n}| = \sqrt{2+2\cos\theta} = 2\sin\frac{\theta}{2}$$

Sol.9 A,B,C,D



PN = $(2\lambda - 1, -3\lambda + 2, 6\lambda - 5)$

$\vec{PN}\ .\ (2, -3, 6) = 0$

$2(2\lambda - 1) + 3(3\lambda + 2) + 6(6\lambda - 5) = 0$

$$\lambda = \frac{26}{49} \qquad \Rightarrow N\left(\frac{52}{49}, \frac{-79}{49}, \frac{156}{49}\right)$$

Equation of PN

$$\frac{x-1}{2\lambda-1} = \frac{y-2}{-3\lambda-2} = \frac{z-5}{6\lambda-5}$$

Put $\lambda = \frac{26}{49}$

$$\frac{x-1}{3} = \frac{y-2}{-176} = \frac{z-5}{-89}$$

Let a point $Q(2\mu, -3\mu, 6\mu)$

PQ will be $\perp^n$ to normal vector of given plane.

$\{(2\mu - 1), (-3\mu - 2), (6\mu - s), (3, 4, 5)\} = 0$

$3(2\mu - 1) + 4(-3\mu - 2) + 5(6\mu - 5) = 0$

$$\Rightarrow \mu = \frac{3}{2}$$

$$Q\left(3, \frac{-9}{2}, 9\right)$$

Equaton of PR

$$\frac{x-1}{2\mu-1} = \frac{y-2}{-3\mu-2} = \frac{z-5}{6\mu-5}$$

Put $\mu = \frac{3}{2}$

$$\frac{x-1}{4} = \frac{y-2}{-13} = \frac{z-5}{8}$$

Sol.10 A,B

$2x - 3y - 7z = 0$

$3x - 14y - 13z = 0$

$8x - 31y - 33z = 0$

Above three planes are passing thorugh origin.

and passes through common line.

Sol.11 B,C

$$\hat{n} = \pm\left(\frac{-3,2,6}{7}\right) = \pm\left(\frac{-3}{7}, \frac{2}{7}, \frac{6}{7}\right)$$

$$-\frac{3x}{7} + \frac{2y}{7} + \frac{6z}{7} = 7$$

$-3x + 2y + 6z - 49 = 0$

and $\quad \frac{3x}{7} - \frac{2y}{7} - \frac{6z}{7} = 7$

$3x - 2y - 6z - 49 = 0$

Sol.12 B,C

Let a point Q $(3\lambda + 15, 8\lambda + 2, -5\lambda + 6)$

PQ = $(2\lambda + 10, 8\lambda - 5, -5\lambda + 3)$

$3(3\lambda + 10) + 8(8\lambda - 5) - 5(-5\lambda + 3) = 0$

$9\lambda + 30 + 64\lambda - 40 + 25\lambda - 15 = 0$

$98\lambda = 35$

$$\lambda = \frac{35}{98} \qquad \Rightarrow PQ = 14 \ \text{(B)}$$

and plane equation $9x - 4y - 14 = 0$